The Study of Some Problems in Fixed Point Theory on Various Spaces

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TITLE OF THE THESIS:

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1. INTRODUCTION /ORIGIN OF PROPOSAL

(Panitchpakdi, 1956)introduce hyper convex metric spaces while studying the problem of uniformly expanding mapping between metric spaces on an ongoing basis. The recent fixed-point interest of hyper convex spaces' theory goes back to(Sine, 1979) results. (Soardi, 1979)Who autonomously verified that non-expansive fixed point (FP) property in bounded hyper convex spaces, mappings hold? As then, several different outcomes have been demonstrated to fit in hyper convex spaces, and very recently, many of them were stated under non compactness conditions (see, for instance (panelD.Bugajewski, 2000)(Es et al., 2001)(Esp´ınola, 1999)(SHIN, 1997).

"The genetic way to show whether or not an equation has a solution is simply to put it as a problem of a fixed point." Thus, we can assume that the problem of finding the fixed points of an m (u) function is the same as the problem of finding the solution of the equation m (v) = 0, where m (u) = M (v) + v. Consider, to explain this fact, the Quadratic v-equation $v^2 - 7v + 6 = 0$. This simple quadratic equation's roots

$v = \frac{v^2+6}{7}$.

By assuming $m(v) = \frac{v^2+6}{7}$,v=1 and v=6 are FP of m, we may assume that the issue of locating the m(v) fixed points is similar to the question of finding out the solution to the equation M(v)=0, where m(u) = M(v) + v. The findings relating to the existence of fixed points are called fixed points. Nevertheless, "many fixed point outcomes in literature" are not Quite constructive since these findings only guarantee the presence of a fixed point, and a fixed point theory does not have any assistance in locating the same one. There are many perspectives and area spaces in FP theory.

FP Theory is categorized into three main approaches:

- Metric FP Theory
- Discrete FP Theory
- Topological FPTheory

Thirty-seven years later,(Horvath 1993) found that space is one of his c-spaces, a vital object of the KKM theoretic study. (Khamsi, 1996)stated the KKM theorem for hyper convex metric spaces, and then there have emerged mostly no. of works on them related to the theory of KKM; see (Park, 2012) and references therein. It is later recognized that the KKM principle of hyper convex metric space effects mostly has a clear impact of much more general results for G-convex spaces because of themselves.

In recent years, many authors have presented interesting generalizations of Metric spaces. In 2018, Mohamed Jleli, Bessem Samet presented the idea of $\mathcal{F}$ -metric space and, while comparing it with metric spaces, they proved that every metric space is an $\mathcal{F}$ -metric space but the converse is not true, confirming that $\mathcal{F}$ -metric space is more general than the metric space. In 2019, N. Manav and D. Turkoglu introduced a new class of generalized metric space called modular $\mathcal{F}$-metric space as a generalization of modular and $\mathcal{F}$ -metric space.

2. BASIC PROPERTIES AND DEFINITIONS

The hyper-convex (M, d) is metric space in which there is only 1 path b/w2 points z & u. It would involve that if w is point b/w z & u, by which we denote if d (z, w) + d (w, u) = d(z, u) then we discern that w is truly on path b/w z & u, this will stimulate next metric interval concept.

Definition 2.1. Suppose (M, d) be MS also assume z, u ∈ M., An arc from z to u, is topological embedding image α: [p, q] → M of closed interval [p, q] of R such that α (p) = z and α (q) = u. A segment of geodesic from z to u is isometric embedding image α: [p, q] → M such that α (p) = z and α (q) = u. The geodesic segment will be known as the metric segment, also represented by [z, u] throughout this work.

Definition 2.2. A MS (M, d) be hyper convex metric space if

$$\bigcap_{\alpha\in\Gamma} B(z_\alpha, t_\alpha) \neq \emptyset$$

For one set of closed balls $\{B(z_\alpha, t_\alpha)\}$ $\alpha \in \Gamma$ like $d(z_\alpha, z_\beta) \leq t_\alpha + t_\beta$, $\alpha, \beta \in \Gamma$. It is not difficult to observe the completion of hyper convex metric spaces. We will constantly use this fact. Whenever the orbits are bound in hyper convex metric spaces $t(z) = \delta(z)/2$. Sets S (z), SS (z) are nonempty since diam S (z) ≤ 2t (z). Furthermore,

$$S(z) = \bigcap_{\alpha \in G} B(F_p z, t(z)).$$

Definition 2.3. Centre of orbit of z ∈ M is set S (z) = {u ∈M: r (u, z) = r (z)} as well as centre of S (z) is denoted by

$$SS(z) = \bigcap_{u \in s(z)} B(u, r(z) \cap S(z))$$

Definition 2.4. Suppose M be hyper convex(HC) space. A mapping F: S → S of subset S of M is supposed to be Lipschitzian, whether there exists non-negative no. R like that d(F_z, F_u) ≤ Rd(z, u) for all z and u ∈ S. Smallest such R is known as k−Lipschitz constant also will be represented by (K). Similar mapping is known as uniformly k−Lipschitzian if sup m≥1 K (F^m) < ∞ respectively sup m≥m0 K (F^m) < ∞ for some $m_0 \geq 1$. Note that uniformly k−lipshitzian mappings require not being continuous. If K(F) ≤ 1, then F is known as non-expansive & ultimately nonexpansive if sup m≥m0 K(F^m) ≤ 1 for several $m_0 \geq 1$. If the map is homogeneously k−lipshitzian, then one may locate an equal distance for which map is non-expansive. Certainly, let F: S → S be homogeneously k−lipshitzian. Setting τ (z, u) = sup {Kd (F^Mz, F^mu) : m = 0, 1, 2, 3 . . . , } for z, u ∈ S.

Theorem 2.5. Let M be bounded MSalso let $\{H_\beta\}\beta \in \Gamma$ be reducing family of non-empty HC subsets of M. After that $\bigcap_{\beta \in \Gamma} H_\beta \neq \emptyset$And it is hyper convex.

Let $\mathcal{F}$ denote the class of functions $f: (0, \infty) \to \mathbb{R}$ which satisfy the following conditions:

$(\mathcal{F}_1) f$ is non-decreasing, i.e., $0 < s < t \Rightarrow f(s) \leq f(t)$.

$(\mathcal{F}_2)$ For every sequence $\{t_n\}_{n \in \mathbb{N}} \subseteq (0, +\infty)$, we have

$$\lim_{n \to +\infty} t_n = 0 \Leftrightarrow \lim_{n \to +\infty} f(t_n) = -\infty$$

The definition of an $\mathcal{F}$ -metric space has been introduced as follows.

Definition 2.6. Let X be a non-empty set and $D: X \times X \to [0, \infty)$ be a given mapping. Suppose there exists $(f, \alpha) \in \mathcal{F} \times [0, \infty)$ such that:

(D1) $D(x, y) = 0 \Leftrightarrow x = y$ for all $(x, y) \in X \times X$.

(D2) $D(x,y) = D(y,x)$ for all $(x,y) \in X \times X$.

(D3) For every $(x,y) \in X \times X$, for each $N \in \mathbb{N}, N \geq 2$ and for every $(u_i)_{i=1}^{N} \subseteq X$ with $(u_1, u_N) = (x,y)$, we have

$$D(x,y) > 0 \Longrightarrow f(D(x,y)) \leq f\left(\sum_{i=1}^{N-1} D(u_i, u_{i+1})\right) + \alpha$$

Then D is said to be an $\mathcal{F}$ -metric on X and the pair (X,D) is said to be an $\mathcal{F}$ -metric space. Hence, the class of all $\mathcal{F}$ -metric spaces contain the class of all metric spaces for any $f \in \mathcal{F}$ and $\alpha = 0$.

Definition 2.7. Let (X,D) be an $\mathcal{F}$ -metric space. A subset C of X is said to be $\mathcal{F}$ -open if for every $x \in C$, there is some $r > 0$ such that $B(x,r) \subset C$ where

$$B(x,r) = \{y \in X : D(y,x) < r\}$$

We say that a subset C of X is $\mathcal{F}$ -closed if $X \setminus C$ is $\mathcal{F}$ -open. The family of all $\mathcal{F}$ -open subsets of X is denoted by $\tau_{\mathcal{F}}$.

Definition 2.8. Let (X,D) be an $\mathcal{F}$ -metric space. Let $\{x_n\}_{n\in\mathbb{N}}$ be a sequence in X. We say that $\{x_n\}_{n\in\mathbb{N}}$ is $\mathcal{F}$ -convergent to $x \in X$ if $\{x_n\}_{n\in\mathbb{N}}$ is convergent to $x \in X$ with respect to the topology $\tau_{\mathcal{F}}$.

Proposition 2.9. Let (X,D) be an $\mathcal{F}$ -metric space. Then, for any nonempty subset A of X, we have

$$x \in \bar{A}, r > 0 \Longrightarrow B(x,r) \cap A \neq \phi$$

Proposition 2.10. Let (X,D) be an $\mathcal{F}$ -metric space. Let $\{x_n\}_{n\in\mathbb{N}}$ be a sequence in X and $x \in X$. Then the following are equivalent:

(i) $\{x_n\}_{n\in\mathbb{N}}$ is $\mathcal{F}$ -convergent to x.

(ii) $D(x_n, x) \to 0$ as $n \to \infty$.

Proposition 2.11. Let (X, D) be an $\mathcal{F}$ -metric space and $\{x_n\}_{n\in\mathbb{N}}$ be a sequence in X. Then

$(x, y) \in X \times X, \lim_{n\to\infty} D(x_n, x) = \lim_{n\to\infty} D(x_n, y) = 0 \Longrightarrow x = y.$

The definition of modular $\mathcal{F}$ -metric as more generalized version of metric, modular metric and $\mathcal{F}$ -metric.

Definition 2.12 Let X be a nonempty set and $F_\lambda: (0,\infty) \times X \times X \to [0,\infty]$ be a function. If there exists $(f, \alpha) \in \mathcal{F} \times [0,\infty)$, such that

(F1) $F_\lambda(x, y) = 0 \Leftrightarrow x = y$, for all $(x, y) \in X \times X$;

(F2) $F_\lambda(x, y) = F_\lambda(y, x)$ for all $(x, y) \in X \times X$;

(F3) For all $(x, y) \in X \times X, p \in \mathbb{N}$ with $p \geq 2$ and for all $(v_i)_{i=1}^{p} \subset X$ with $(v_1, v_p) = (x, y)$, we have;

$$F_\lambda(x, y) > 0 \text{ implies } f(F_\lambda(x, y)) \leq f\left(\sum_{j=1}^{p-1} F_{\frac{1}{j}}(v_j, v_{j+1})\right) + \alpha$$

then F_λ is called an modular $\mathcal{F}$ -metric on X. The pair (X, F_λ) is called an modular $\mathcal{F}$ -metric space.

Now, we start to built a topology on modular $\mathcal{F}$ -metric space by definitions step by step. Very nice work will lead idea of building topology. The first step is giving boundary conditions for modular $\mathcal{F}$ -metric.

Definition 2.13 Let X be a nonempty set and $F_\lambda: (0,+\infty) \times X \times X \to [0,\infty)$ be a mapping that satisfies $(D1)$ and (D2) with recpect to $(f, \alpha) \in \mathcal{F} \times [0,\infty)$, than we say that (X, F_λ) is modular $\mathcal{F}$ -metric bounded space, if there is a metric d_ω on X such that;

$$(x, y) \in X \times X, F_\lambda(x, y) > 0 \text{ implies } f(d_\omega(x, y)) \leq f(F_\lambda(x, y)) \leq f(d_\omega(x, y)) + \alpha$$

If we have modular $\mathcal{F}$ -metric with (f, α), then we have bounded modular $\mathcal{F}$ -metric with (f, α) as below:

Theorem 2.14 Let X be a nonempty set, and let $F_\lambda: (0,+\infty) \times X \times X \to [0,\infty)$ e a given mapping satisfying $(D1)$ and $(D2)$. Let $(f,\alpha) \in \mathcal{F} \times [0,\infty)$ and suppose that f is continuous from the right. Then the following statements are equivalent:

(i) (X,F_λ) is an modular $\mathcal{F}$ -metric on X with (f,α) defined above.

(ii) (X,F_λ) is an modular $\mathcal{F}$ -metric bounded on X with recpect to (f,α).

Very important to define what is open(closed) set(ball) on a topology. Which conditions are necessary for this definition and if we have any set how to determine a closure of a set, how to find a convergent sequence in the space, and so on.

Definition 2.15 Let(X,F_λ) is a modular $\mathcal{F}$ -metric space and let $U \subset X$ is a nonempty subset on it. $U \subset X$ is called modular F_λ -open, if for every $x \in U$, there is some $r > 0$ such that $B_{F_A}(x,r) \subset U$ where $B_{F_\lambda}(x,r) = \{y \in X: F_\lambda(x,y) < r\}$. Then, if $X - U = F$ is modular F_λ -open subset, $F \subset X$ is called modular F_λ -closed. For the family of all modular F_λ -open subsets of X we determine τ_{F_A}.

Proposition 2.16 Let (X,F_λ) be a modular $\mathcal{F}$ -metric space. Then τ_{F_λ} is a topology on X.

Proposition 2.17 Let (X,F_λ) be a modular $\mathcal{F}$ -metric space. Then, for any nonempty $A \subset X$, the following statements are equivalent:

(i) A is modular F_λ-$.closed$

(ii) For any sequence $\{x_n\}_{n\in\mathbb{N}} \subset A$ we have $\lim_{n\to\infty} F_\lambda(x_n,x) = 0, x \in X$ implies $x \in A$.

Definition 2.18 Let (X,F_λ) be a modular $\mathcal{F}$ -metric space. Let a nonempty subset $A \subset X$. We denote by $\bar{A}$ the closure of A with respect to the topology τ_{F_1}, i.e. $\bar{A}$ is the intersection of all modular $\mathcal{F}$ -metric closed subsets of X containing A. Clearly, A is the smallest modular $\mathcal{F}$ -metric closed subset which contains itself.

Proposition 2.19 Let (X,F_λ) be a modular $\mathcal{F}$ -metric space. Then, for any nonempty subset $A \subset X$, we have $x \in \bar{A}, r > 0$ implies $B_{F_A}(x,r) \cap A \neq \emptyset$.

Definition 2.20 Let (X, F_λ) be a modular $\mathcal{F}$ -metric space. Let $\{x_n\} \subset X$ be a sequence in X. We say that $\{x_n\} \subset X$ is modular $\mathcal{F}$ -convergent to point $x \in X$, if $\{x_n\} \subset X$ is convergent to x with respect to the topology τ_{F_λ}, i.e. for every modular $\mathcal{F}$ -open subset $U_x \subset X$ containing x, there exists some $n \geq N, N \in \mathbb{N}$ such that $x_n \in U_x$, for all n. Then, $\lim_{n\to\infty} F_\lambda(x_n, x) = 0, x \in X$ implies x is the limit of $\{x_n\} \subset X$

Proposition 2.21 Let (X, F_λ) be a modular $\mathcal{F}$ -metric space. Let $\{x_n\} \subset X$ be a sequence in X, and $x \in X$. The following statements are equivalent:

(i) $\{x_n\} \subset X$ modular $\mathcal{F}$ -convergent to $x \in X$.

(ii) $\lim_{n\to\infty} F_\lambda(x_n, x) = 0$.

Now, let us define modular $\mathcal{F}$ -Cauchy sequence, then completeness definition and conditions for modular $\mathcal{F}$ metric space.

Definition 2.22 Let (X, F_λ) be a modular $\mathcal{F}$ -metric space. Let $\{x_n\} \subset X$ be a sequence in X.

(i) We say that $\{x_n\} \subset X$ is modular $\mathcal{F}$ -Cauchy, if $\lim_{n,m\to\infty} F_\lambda(x_n, x_m) = 0$.

(ii) We say that (X, F_λ) modular $\mathcal{F}$ -complete, if every modular $\mathcal{F}$ -Cauchy sequence is modular $\mathcal{F}$ -convergent to a certain element in X.

Proposition 2.23 Let (X, F_λ) be a modular $\mathcal{F}$ -metric space. If $\{x_n\} \subset X$ is modular $\mathcal{F}$ -convergent, then it is modular $\mathcal{F}$ - Cauchy.

The definiton of modular $\mathcal{F}$ -compact set is giving more details about the topology on modular $\mathcal{F}$ -metric space.

Definition 2.24 Let (X, F_λ) be a modular $\mathcal{F}$ -metric space. Let $C \subset X$ be a nonempty subset. We say that C is modular $\mathcal{F}$ -compact if C is compact with respect to the topology τ_{F_λ} on X.

Proposition 2.25 Let (X, F_λ) be a modular $\mathcal{F}$ -metric space. Let C be a nonempty subset of X. Then, the following statements are equivalent:

(i) C is modular $\mathcal{F}$ -compact.

(ii) For any sequence $\{x_n\} \subset C$ there exist a subsequence $\{x_{n_k}\} \subset \{x_n\}$ and $x \in A$ such that ;

$$\lim_{k\to\infty} F_\lambda(x_{n_k}, x) = 0$$

Definition 2.26 Let (X, F_λ) be a modular $\mathcal{F}$ -metric space. Let $C \subset X$ be a nonempty subset. The subset C is called modular sequentially $\mathcal{F}$ -compact, if for any sequence $\{x_n\} \subset C$, there exist a subsequence $\{x_{n_k}\} \subset \{x_n\}$ and $x \in C$ such that

$$\lim_{k\to\infty} F_\lambda(x_{n_k}, x) = 0.$$

Definition 2.27 Let (X, F_λ) be a modular $\mathcal{F}$ -metric space. Let $C \subset X$ be a nonempty subset. The subset C is called modular $\mathcal{F}$ -totally bounded, if for any $r > 0$ there exists a sequence $(x_j), j = 1,2, \dots, n \subset C$ such that $C \subset \cup B_{F_\lambda}(x_j, r)$.

Proposition 2.28 Let (X, F_λ) be a modular $\mathcal{F}$ -metric space. Let $C \subset X$ be a nonempty subset. Then

(i) C is modular $\mathcal{F}$ -compact $\Leftrightarrow$ C is modular sequentially $\mathcal{F}$ -compact.

(ii) C is modular $\mathcal{F}$ -compact $\Rightarrow$ C is modular $\mathcal{F}$ -totally bounded.

3. RATIONALE/HYPOTHESIS

- A vibrant geometric structure is given by the hyper convexity, which is not based on the underlying linear system.
- We start from this section by stating that there is no direct relationship b/w linear convexity & metric hyper-convexity when working with linear hyper-convex space.
- We deduce various types of fixed point theorems in F-metric spaces and modular F-metric spaces.
- We must recall the many stages of modular F-metric space and F-metric space extension to various abstract modular F-metric space and F-metric space subclasses. These numerous generalizations of modular F-metric space and F-metric spaces and to have a better understanding of their properties.

4. AIM AND OBJECTIVES

- To generalize and develop established FP theorems by implementing the selection theorem in hyper convex spaces.
- To study the basic structure of hyperconvex spaces.
- To prove FP theorems uniqueness for uniformly k−Lipschitzian mappings in HC metric spaces.
- To introduce many such generalizations of hyper convex metric spaces and deepen the understanding of their nature.
- To applying the ensemble method to generalized the hyper convex mechanism in metric space.
- To optimize the development's exposition during the last decade of metric fixed point theory on hyper convex metric spaces.
- To study the basic structure of modular F-metric spaces and F-metric spaces.
- To prove a new common fixed point theorem for four mappings in modular F-metric spaces and gives its results.
- To obtain fixed points as well as common fixed-point results for Reich-type F-contractions for both single and set-valued mappings in modular F-metric spaces.
- To prove various types of fixed point theorems in F-metric spaces and modular F-metric spaces.
- To establish a new version of the Banach contraction principle in the setting of modular F-metric spaces.
- To provide the proof of the metrizability of modular F-metric spaces.
- To prove some Fixed-Point Theorems over a generalized modular $\mathcal{F}$ -metric spaces.
- To generalize Banach Contraction Principle in various metric spaces.
- To introduce different examples in various metric spaces.
- To prove Common Fixed Point Results on Modular $\mathcal{F}$ -Metric Spaces.

5. REVIEW OF LITERATURE

(**N. Manav and D. Turkoglu, 2019**)Introduced a new class of generalized metric space called modular $\mathcal{F}$-metric space as a generalization of modular. They compare the topology produced

by modular metric and by modular $\mathcal{F}$ -metric, then cover some useful properties of this topology for fixed point theorems for future studies. In the end, they prove Banach contraction principle for modular $\mathcal{F}$ -metric space. They give the definition of modular $\mathcal{F}$-metric space which is more general then $\mathcal{F}$-metric space, modular metric space, definition of modular $\mathcal{F}$-convergent, modular $\mathcal{F}$ -cauchy, modular $\mathcal{F}$- compact and establish an equivalent relationship between modular $\mathcal{F}$-metric spaces and modular F-matric bounded space.

(Đekić & Bin-Mohsin, 2019) Consider different findings in the context of B-metric spaces for convex contraction mappings. The recent results published in convex contractions defined in standard metric spaces are generalized, expanded, corrected, and enriched, among other things, to supposed B-metric spaces. An explanation indicates that this generalization is genuine. We must remember that this work is just beginning our analysis of proprietors of convex contractions observed in any metric space. Our considerations apply in articles to be published to cone metrics spaces, G-metrics, partial metric spaces, expanded B-metrics, GB-metrics, also many more.

(Wang et al., 2019) Checked validity in certain FP theorems metric space of partial cone. Any FP theorems of partial metric space are generalized by sequences & iteration into partial cone metric space, thus achieving new FP results. Any FP theorems range from metric part space & cone space into metric partial cone space. Conditions of the FP theorem can be weakened, particularly for multi-valued mappings. For example, in this article, for example. Some concerns require further review.

(Li & Wu, 2019) Study in the general setting of several bit quantizes mentioned above design problem. We illustrate that solutions to 1^{st} order criterion for optimality can be accomplished beyond the workable threshold as FP on specific nonlinear maps. The existence & uniqueness of FP was defined by using FP theorems of Schauder & Brouwer. Finally, the fixed point obtained indicates that the global optimum quantization thresholds are generated. As far as we are aware, this work is the first in which FP theories address SNR quantization problems strictly in wireless relay communication. An optimal architecture dilemma was described to reduce the system bit error rate. The quantization threshold measured with a proposed rule of thumb was considered in only one binary quantization case.

(**Jleli et al., 2018**)Presented the idea of $\mathcal{F}$ -metric space and, while comparing it with metric spaces, they proved that every metric space is an $\mathcal{F}$ -metric space but the converse is not true, confirming that $\mathcal{F}$ -metric space is more general than the metric space. The authors made use of a certain class of auxiliary functions to coin the idea of such abstract spaces.With the help of concrete examples, they obtained a similar result for s-relaxed metric space.They proved various types of fixed point theorems in F-metric spaces.

(Weirich et al., 2018)suggest operator who can estimate the function of logarithm at base e as an operator is executed by approximation of Taylor Series with five terms. We also give algo liable for allocating p values from particular integrated into the Taylor approach convergence area. By co-simulation using both ModelSim ® and Matlab ®, this was probably to define below 0.6 percent for the complete i/p range. I/p range utilized in this work is in numeral format by one-byte length, as well as system o/p has 19 bits by 4.15 form.

(Nguyen et al., 2018)the proposed processor has cosine, sine, hyperbolic of cosine, hyperbolic of sine, & multiplication fun. Capable of operation. CMOS hardware architecture can operate at 100 MHz & consumes 12.96 MW of power, and is incorporated in a 180 nm CMOS technology. The architecture is an acceptable alternative for high-performance low-energy applications relative to some previous work.

(Karapınar et al., 2016) Presents generalized contractions in complete -metrical spaces setting also define new FP theorems for this contraction. The paper results strengthen, expand, and unify those well-known results. At last, we set an illustration to demonstrate them.

(Roughgarden & Weinstein, 2016) Providing a solution to the renowned open issue of identifying well-built lower limits on the approximate computation of Nash equilibrium touch complexity. We show that our lower bound (by a total constant factor) FP calculation's much 'smoother' version would suggest: that deterministic two-party communication complexity of finding al= (1/log2N)-approximate Nash equilibrium in N to N bimatrix game is at least N, γ for several constant $\gamma > 0$. (This problem is only O ($\log^6 N$) because of its nondeterministic coordination complexity). ; Deterministic finding difficulty of multi-party communication = ?? define the above question in this paper.

(Agarwal et al., 2016)proved that each non-expansive mapping has FP in convex hyper convex metric space. Various fixed-point theories and generalizations for contractive mappings in MS. 2 sided convex contraction mapping & formed two convex Hardy & Rogers's contraction mapping respectively are added. The work shows that the fixed points of these recently implemented forms two convex contraction mappings are unique in that they use the orbital consistency of mappings at a certain point. Embedded applications typically have strict cost, power consumption, and real-time constraints. Fixed-point arithmetic is now mainstream to be used in embedded systems. The fundamental goal of the DEFIS French ANR Project is, therefore, to provide a complete design flow to refine complex applications at a fixed point. This tool flow is analogous to the lack of interaction between high-level design specification tools and low implementation.

(Le Maire et al., 2016) Reassess pertinence of floating-point computing functions in the FP position. Multiple variants of the logarithm of double-precision function are introduced & tested for this. The dilemma is conceived as FP one after the (classically) range has been decreased. After that, 64-bit integers give far higher consistency than 53-bit mantissa, helping to speed up an evaluation. In the end, the Arithmetic of Multi-Word is both faster and accepted natively in fixed-point applications that are key to a specific implementation. Thanks to all of these, the correctly circled double precise logarithm's pure integer implementation exceeds the prior state-of-the-art, through worst-case execution times being decreased by a factor of 5. This study also presents logarithm variants that input FP no. and o/p outcome in FP. These are both more precise& effective for certain purposes than conventional floating-point-functions. Elementary functions from i/p and o/p floating-point numbers of the mathematical library. However, you can use integer / fixed-point arithmetic to either execute them. Between 1985 and 2005, this alternative wasn't attractive since conventional hardware support 64-bit floating points, however only a 32-bit integer. It has distorted in current years, mainly by widespread support of native 64-bit integer.

Table 1: Research Gap of Fixed Point in Hyper convex Matric Space, modular $\mathcal{F}$ - metric spaces, $\mathcal{F}$ -metric spaces

Year	**Author**	**Title of Research Paper**	**The idea in Research Paper**
2015	(Fathollahi et al., 2015)	Performance comparison of some iteration approaches relating contractive mappings on the convergence rate	To illustrate the explanation for the presence of the previous discrepancy between the points of view.
2016	(Karahan, 2016).	Generalized mixed equilibrium setback with strong & weak convergence set theory for split equality	A novel technique to obtain tough and weak convergence algos for the generalized miscellaneous equilibrium issue of split equality.
2017	(Mbarki & Oubrahim, 2017)	Probabilistic b-metric spaces and nonlinear contractions	Probabilistic feature to known *b*-metric spaces
2018	(Manav, 2018)	Theorems of fixed points in a new type of	A modern notion of widespread modular metric space

		modular metric space	
2018	(Jleli et al., 2018)	On a new generalization of metric spaces	Establish a new version of the Banach contraction principle in the setting of $\mathcal{F}$ -metric spaces
2019	(Okeke et al., 2019)	Random fixed point concepts applied to a random nonlinear integral Hammerstein form equation in Banach spaces	Defining a different new operator called the φ-weakly contraction generalized rational form
2019	(Ayari et al., 2019)	For special generalized proximal β-quasi contractive mappings, the strongest proximity point theorem	To obtain the greatest proximity point outcome for a novel class of non-self mappings T: A?? BT: A??

2019	(N. Manav and D. Turkoglu, 2019)	Common fixed point results on modular F - metric spaces	Compare the topology produced by modular metric and by modular $\mathcal{F}$ -metric
2020	(Harrak & Ahmed, 2020)	Common FP theorems for 2 and 3 mappings	FrequentFPapproaches for 2 and 3 commuting mappings
2020	(Chidume et al., 2020)	A hybrid inertial algorithm to estimate the solution of problems of convex viability in applications	An inertial simulation result for approximating a point in the maximum monotone operator's zeros set for a non-expansive family of operators.

6. PROBLEM STATEMENTS

- The known theorems of a fixed point in hyper convex spaces have not generalized and improved solutions by applying any selection theorem. It gives vague values that are not clear to understand.
- Common Fixed Point Results on Modular $\mathcal{F}$ -Metric Spaces have not proved.
- The computational task of locating fixed non-expansive mapping points under uniformly hyper convex conditions because of the computational difficulty of the difficulty of obtainingFP of non-expansive mappings inconsistently in hyper convex matrics spaces.
- Generalize Banach Contraction Principle in various metric spaces have not proved.

- The complexity of the computational problem of finding convergence rates for a huge class of fixed-point iterations that the dilemma of finding convergence rates already on the unit interval is equal to the limit operator.
- Problem to find a new common fixed point theorem for four mappings in modular F-metric spaces and gives its results.
- The proof of the metrizability of modular F-metric spaces have not proved.
- To obtain fixed points as well as common fixed-point results for Reich-type F-contractions for both single and set-valued mappings in modular F-metric spaces.
- Establish a new version of the Banach contraction principle in the setting of modular F-metric spaces.
- Various types of fixed point theorems in F-metric spaces and modular F-metric spaces have not proved.

7. RESEARCH METHODOLOGY

The notion of the compact and natural structure proposed by JP is applied to hyper convex relation systems. We are penetrating metric spaces, which prove that each commuting family of relation homomorphisms has a common FP for involutive and reflexive. The evidence depends on a clever argument that similar results can be made in case of bounded HC metric spaces and then refined to metric spaces in compact and normal structure from the first author. Since non-extensive mapping is FP hyper convex mapping, we prove that it extends to graphs of reflexively oriented, bounded fixed-point length theorem. This illustrates how hyper convexity is considered in space systems of fixed point matrices or generalized metric spaces. Apply different approach to prove Common Fixed Point Results on Modular $\mathcal{F}$ -Metric Spaces and $\mathcal{F}$ -Metric Spaces. Using different methods to establish a new version of the Banach contraction principle in the setting of modular F-metric spaces. Various types of fixed point theorems in F-metric spaces and modular F-metric spaces can be prove by different approach.

7.1 Study Design

We introduce a typical subclass of metrix spaces called the Horvath spaces, including cspaces. We show hyperbolic metric spaces, metric spaces with continuous midpoints, metric spaces with global NPC (Non-Positive Curvature) & convex hull finite property (CHFP), certain Riemannian

manifolds, B-spaces as relatively new examples of Horvath spaces. Many of their properties are introduced by following our previous works.

7.2 Descriptive Designs

- Cross-sectional study or survey of metric FP theory in various spaces.
- Generalization study of hyper convex, modular $\mathcal{F}$ -metric spaces, $\mathcal{F}$ -metric spaces with respect to FP theory,
- Application of FP theory-

1. Banach Contraction Theorem
2. Caristi's FP Theorem
3. Caristi–Kirk's FP Theorem
4. Clarke's FP Theorem

Proposed optimization approach with fixed point theorem

- Designs Prospective Study
- Experimental Designs

1. We collected a large number of references on hyper convex metric spaces, modular $\mathcal{F}$ -metric spaces, $\mathcal{F}$ -metric spaces related to the fixed point theory.
2. The authors have attempted over the years to the widespread classical idea of distance. They also attempted to confine smetric space's topological characteristics several common results for metric spaces. E.g., several attempts have been made in simplified metric spaces to show an analogy to the classical hyper convex principle. Although some attempts to provide a deep understanding of these findings were successful, other attempts as machine-production publications were not. We assume that one of these publications generates machines as the cone metric space.

7.3 Sampling

There are various sampling methods, such as simple random, systemic, stratified sampling, cluster sampling, etc. Please ensure that the sample size is appropriate for generating reliable data. We derived widespread variants of minimal inequality of Ky Fan, minimax theorem of von Neumannion, the intersection principle of von Neumann-Fan, the analytical alternative of fan

kind, and the theorem of the Nash equilibrium. Therefore, certain historically known instances of the same type are unified and generalized.

7.4 Data Analysis

FP's theory is one of the most effective & influential tools of contemporary mathematics and is the main nonlinear analysis subject. For many mathematicians in the previousfifty years, FP theory has been a prosperous field of study. Theory origins that date back to the late 19th century depend on successive approaches, particularly to differential equations, to determine the presence and uniqueness of solutions. Many prominent mathematicians, such as Cauchy, Fredholm, Lowville, Lipchitz, Peano, and Picard, are concerned with this procedure. It should be remembered that Banach's abstract formulation is credited as the basis for metric FP theory. Before Felix Browder made a significant contribution to establishing nonlinear functional analysis as an active & critical field in mathematics, the theory did not gain enough traction. Distinguished authors have written several excellent books, monographs, and surveys of fixed points theory in the past several years. This monograph aims to discuss certain of its facets until the theory is mature enough to analyze its core themes. One of these monographs aims is to include a readily available autonomous document for the mathematical community to introduce the subject and its progress. This monograph contains eight chapters on various features of the theory of FPs, namely: metric theory of FP, a theorem of Ekeland's variation, hyper convex metric spaces, modular $\mathcal{F}$ -metric spaces, $\mathcal{F}$ -metric spaces discrete point theory, modular functional spaces, a topological theory of FP as well, as a fixed-point theoretical theory of iterative methods.

8. PLAN OF WORK

It includes phase-wise distributions of the work plan, the technical definition, and the standard references (details of methods) of each protocol/study/scheme/process.

Table 2: Plan for our work

Chapterization	**Work Plan**
1	Introduction to metric fixed point theory in various spaces
2	Review of Literature/State of the art methods, Recalls the several stages of extensions of • hyper convex metric spaces, • modular metric spaces, • $\mathcal{F}$ -metric spaces • modular $\mathcal{F}$ - metric spaces to various subclasses of abstract spaces
3	Proposed Methodology/ Devotes to metric abstract hyper convex spaces and their basic properties
4	Result and Discussion of hyper convex metric spaces and different type of fixed point theory
5	Proposed Methodology/ Devotes to metric abstract $\mathcal{F}$ metric spaces and their basic properties
6	$\mathcal{F}$ metric spaces and fixed point theorems
7	Proposed Methodology/ Devotes to metric abstract modular $\mathcal{F}$ metric spaces and their basic properties
8	Fixed point theory and modular $\mathcal{F}$ metric spaces

9	Common fixed-point theory and modular $\mathcal{F}$ metric spaces
10	Conclusion and Future work

9. EXPECTED OUTCOMES

The facts' biggest expected findings would be at least to generalize the existence of non-expansive retractions to a finite dimension smooth and convex computerized in ensemble hyper convex matrices FP spaces. We can also provide more useful insights into both fixed-point theory and computability theory. We can prove that any commuting family with relational HC space has shared FP for involutive and reflexive space. For a convex complete metric space, we can research certain theorems for self-mapping that follow certain contraction principles. Also, a hyper convex plane operator is investigated by several standard theorems for fixed points. Finally, some new findings have been strengthened and expanded.

10. FUTURE SCOPE

We expect that our work's outcome will benefit researchers and enhance and encourage more research into soft metric space and Fuzzy soft metric space for its real-life use. These findings can be generalized in any direction and can also be generalized to non-expansive ensemble method mappings with fixed point theory. Such proven outcomes lead to various approaches and facets of the theory of metric fixed points. All results can be extended in other spaces in the future, e.g., metric space outcomes can be modified as in extended MS, pseudo-MS, compact MS, and ultra metric space. In the future, other researchers may be used, and other conditions may be generalized. Our future work will focus on studying the fixed points of multi-valued and fuzzy mappings in modular $\mathcal{F}$ -metric spaces, with fractional differential inclusion problems as applications.

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